

Towards a NNLO calculation in hadronic heavy hadron production

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1 Introduction

The full next-to-leading order (NLO) corrections to the hadroproduction of heavy flavors have been completed in 1988 [1, 2]. They have raised the leading order (LO) estimates [3] but were still below the experimental results (see e.g. [4]). In a recent analysis theory moved closer to experiment [4]. A large uncertainty in the NLO calculation results from the freedom in the choice of the renormalization and factorization scales. The dependence on the factorization and renormalization scales is expected to be greatly reduced at next-to-next-to-leading order (NNLO). This reduces the theoretical uncertainty. Furthermore, one may hope that there is yet better agreement between theory and experiment at NNLO.

In Fig. 1 we show one generic diagram each for the four classes of contributions that need to be calculated for the NNLO corrections to hadroproduction of heavy flavors. They involve the two-loop contribution (Fig. 1a), the loop-by-loop contribution (Fig. 1b), the one-loop gluon emission contribution (Fig. 1c) and, finally, the two gluon emission contribution (Fig. 1d). An interesting subclass of the diagrams in Fig. 1c are those diagrams where the outgoing gluon is attached directly to the loop. One then has a five-point function which has to be calculated up to $\mathcal{O}(\varepsilon^2)$.

In our work we have concentrated on the loop-by-loop contributions exemplified by Fig. 1b. Specifically, working in the framework of dimensional regularization, we have calculated $\mathcal{O}(\varepsilon^2)$ results for all scalar one-loop one-, two- and three-point integrals that are needed in the calculation of hadronic heavy

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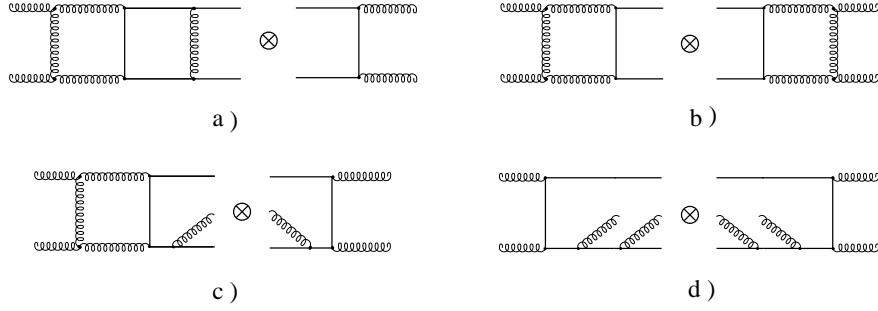


Fig. 1. Exemplary diagrams for the NNLO calculation of heavy hadron production

flavour production. Work on the relevant four-point integrals is in progress. The integrations were generally done by writing down the Feynman parameter representation for the corresponding integrals, integrating over Feynman parameters up to the last remaining integral, expanding the integrand of the last remaining parametric integral in terms of ε and doing the last parametric integration on the coefficients of the expansion. Because the one-loop integrals exhibit infrared (IR)/collinear (M) singularities up to $\mathcal{O}(\varepsilon^{-2})$ one needs to know the one-loop integrals up to $\mathcal{O}(\varepsilon^2)$ because the one-loop contributions appear in product form in the loop-by-loop contributions. It is clear that the spin algebra and the calculation of tensor integrals in the one-loop contributions also have to be done up to $\mathcal{O}(\varepsilon^2)$. This work is in progress.

Due to lack of space we can only present a few exemplary results. Since the four-point functions are the most difficult we concentrate on them.

2 Four-point functions

As a sample calculation we discuss the $(0, 0, m, 0)$ -box with one massive propagator depicted in Fig. 2. As explained before one needs to calculate each

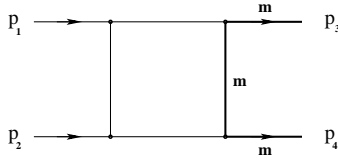


Fig. 2. One-loop box with one internal massive propagator

one-loop integral up to $\mathcal{O}(\varepsilon^2)$ in order to obtain the finite terms in the loop-by-loop contributions. The box integral Fig. 2 is represented by the integral

$$D(-p_2, p_4, p_3, 0, 0, m, 0) = \mu^{2\varepsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2)(q-p_2)^2[(q-p_2+p_4)^2-m^2](q-p_2+p_4+p_3)^2},$$

where p_1, p_2, p_3 and p_4 are external momenta with $p_1^2 = p_2^2 = 0, p_3^2 = p_4^2 = m^2$ and $n = 4 - 2\varepsilon$ is the dimension of space-time.

The $\varepsilon^{-2}, \varepsilon^{-1}$ and ε^0 coefficients have been known for some time [1, 2] and will not be listed here. We define Mandelstam-type variables by $s \equiv (p_1 + p_2)^2, t \equiv (p_1 - p_3)^2 - m^2$ and $\beta = (1 - 4m^2/s)^{1/2}, x = (1 - \beta)/(1 + \beta)$.

For the real part of the $\mathcal{O}(\varepsilon)$ term we obtain:

$$\begin{aligned} & \frac{iC_\varepsilon(m^2)}{12st} \varepsilon \left[6 \ln^3 \frac{s}{m^2} + 20 \ln^3 \frac{-t}{m^2} + \ln^3 x + 6 \ln^2 x \ln \frac{-(s+2t-s\beta)}{2m^2} \right. \\ & - 12 \ln x \ln^2 \frac{-(s+2t-s\beta)}{2m^2} + 8 \ln^3 \frac{-(s+2t-s\beta)}{2m^2} \\ & - 12 \ln^2 \frac{-t}{m^2} \left(2 \ln(-1 - \frac{t}{m^2}) + 3 \ln x - \ln \frac{-(s+2t-s\beta)}{2m^2} + 4 \ln \frac{s+2t+s\beta}{2m^2} \right) \\ & - 3 \ln^2 \frac{s}{m^2} \left(6 \ln \frac{-t}{m^2} + 3 \ln x + 2 \ln \frac{-(s+2t-s\beta)}{2m^2} + 4 \ln \frac{s+2t+s\beta}{2m^2} \right) + 12 \ln \frac{s}{m^2} + \\ & 24 \ln \frac{-t}{m^2} + 24 \ln \frac{s}{m^2} \ln \frac{-t}{m^2} + 48 Li_3(\frac{m^2+t}{t}) + 48 Li_3(1 + \frac{t}{m^2}) + 24 Li_3(-x) - \\ & 24 Li_3(\frac{-2m^2+t(-1+\beta)}{t(1+\beta)}) - 48 Li_3(\frac{2m^2+t-t\beta}{2m^2}) + 24 Li_3(\frac{2m^2+t-t\beta}{m^2(1+\beta)}) \\ & - 24 Li_3(\frac{2m^2+t-t\beta}{t-t\beta}) - 48 Li_3(\frac{2m^2}{2m^2+t+t\beta}) + 24 Li_3(-\frac{m^2(-1+\beta)}{2m^2+t+t\beta}) \\ & - 24 Li_3(\frac{t(-1+\beta)}{2m^2+t+t\beta}) - 24 Li_3(\frac{2m^2+t+t\beta}{t+t\beta}) - 6 \ln \frac{-t}{m^2} \left(4 + 2 \ln^2(-1 - \frac{t}{m^2}) - \ln^2 x - \right. \\ & 4 \ln x \ln \frac{-(s+2t-s\beta)}{2m^2} + 6 \ln^2 \frac{-(s+2t-s\beta)}{2m^2} \\ & \left. - 4 \ln(-1 - \frac{t}{m^2}) \left(\ln x + \ln \frac{-(s+2t-s\beta)}{2m^2} + \ln \frac{s+2t+s\beta}{2m^2} \right) + 8 Li_2(1 + \frac{t}{m^2}) \right. \\ & + 8 Li_2(\frac{t(-1+\beta)}{2m^2}) + 4 Li_2(\frac{m^2(-1+\beta)}{(m^2+t)(1+\beta)}) - 4 Li_2(\frac{(m^2+t)(-1+\beta)}{m^2(1+\beta)}) \\ & + 8 Li_2(\frac{2m^2}{2m^2+t+t\beta}) - 12 \zeta(2) \left. \right) - 12 \ln x \zeta(2) + 48 \ln \frac{-(s+2t-s\beta)}{2m^2} \zeta(2) - 6 \ln \frac{s}{m^2} \left(2 + \right. \\ & 4 \ln^2 \frac{-t}{m^2} - \ln^2 x - 2 \ln x \ln \frac{s+2t+s\beta}{2m^2} - 4 \ln \frac{-t}{m^2} \left(-1 + \ln x + \ln \frac{-(s+2t-s\beta)}{2m^2} \right) + \\ & \left. 2 \ln \frac{s+2t+s\beta}{2m^2} \right) - 4 Li_2(\frac{t(-1+\beta)}{2m^2}) - 4 Li_2(\frac{2m^2}{2m^2+t+t\beta}) + 2 \zeta(2) \left. \right] - 36 \zeta(3) \Big], \\ & \text{where } C_\varepsilon(m^2) \equiv \frac{\Gamma(1+\varepsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{m^2} \right)^\varepsilon. \end{aligned}$$

One also needs the imaginary part of the $(0,0,m,0)$ -box since the total contributions from the loop-by-loop contribution contains also imaginary parts via $|A|^2 = (ReA)^2 + (ImA)^2$. Note, however, that the imaginary part is only needed up to $\mathcal{O}(\varepsilon)$ since the IR/M singularities in the imaginary parts of the one-loop contributions are of $\mathcal{O}(\frac{1}{\varepsilon})$ only. For the $\mathcal{O}(\varepsilon)$ absorptive (imaginary) part we obtain:

$$\begin{aligned} & -\frac{iC_\varepsilon(m^2)}{4st} \varepsilon \pi \left[3 \ln^2 \frac{s}{m^2} - 4 \ln \frac{s}{m^2} \ln \frac{-t}{m^2} - 8 \ln^2 \frac{-t}{m^2} - 2 \ln \frac{s}{m^2} \ln x + 4 \ln \frac{-t}{m^2} \ln x + \right. \\ & \left. 3 \ln^2 x - 4 \ln \frac{s}{m^2} \ln \frac{-(s+2t-s\beta)}{2m^2} - 4 \ln x \ln \frac{-(s+2t-s\beta)}{2m^2} + 4 \ln^2 \frac{-(s+2t-s\beta)}{2m^2} - \right. \end{aligned}$$

$$4 \ln \frac{s}{m^2} \ln \frac{s+2t+s\beta}{2m^2} + 8 \ln \frac{-t}{m^2} \ln \frac{s+2t+s\beta}{2m^2} + 4 \ln x \ln \frac{s+2t+s\beta}{2m^2} + 8 Li_2\left(\frac{t(-1+\beta)}{2m^2}\right) + 8 Li_2\left(\frac{2m^2}{2m^2+t+t\beta}\right) + 8 \zeta(2) \Big].$$

The ε^2 -results for the $(0, 0, m, 0)$ -box are too lengthy to fit into this report. They will be presented in a forthcoming publication [5]. A new feature of the ε^2 -contributions is that the result can no longer be expressed in terms of logarithms and polylogarithms. They involve more general functions - the multiple polylogarithms introduced by Goncharov in 1998 [6]. A multiple polylogarithm is represented by

$$Li_{m_k, \dots, m_1}(x_k, \dots, x_1) = \int_0^{x_1 x_2 \dots x_k} \left(\frac{dt}{t} \circ \right)^{m_1-1} \frac{dt}{x_2 x_3 \dots x_k - t} \circ \left(\frac{dt}{t} \circ \right)^{m_2-1} \frac{dt}{x_3 \dots x_k - t} \circ \dots \circ \left(\frac{dt}{t} \circ \right)^{m_k-1} \frac{dt}{1-t},$$

where the iterated integrals are defined by

$$\int_0^\lambda \frac{dt}{a_n - t} \circ \dots \circ \frac{dt}{a_1 - t} = \int_0^\lambda \frac{dt_n}{a_n - t_n} \int_0^{t_n} \frac{dt_{n-1}}{a_{n-1} - t_{n-1}} \times \dots \times \int_0^{t_2} \frac{dt}{a_1 - t_1}.$$

Besides the scalar $(0, 0, m, 0)$ -box one also needs to calculate the scalar $(0, 0, m, m)$ and $(m, m, 0, m)$ -boxes. Work is in progress on the calculation of these boxes.

3 Summary

We have reported on the results of an ongoing calculation of the ε -expansion of the scalar one-loop integrals up to $\mathcal{O}(\varepsilon^2)$ that are needed for the NNLO calculation of hadronic heavy hadron production. In order to arrive at the full amplitude structure of the one-loop contributions one still has to include the ε -dependence resulting from the Passarino-Veltman decomposition of the tensor integrals, and the ε -dependence of the spin algebra calculation. Putting all these pieces together one might optimistically say, when considering the four classes of diagrams Fig. 1, that the present calculation constitutes one-fourth of the full NNLO calculation of hadronic heavy hadron production.

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